Nash Equilibrium - definition

- A mixed-strategy profile σ^* is a Nash equilibrium (NE) if for every player *i* we have $u_i(\sigma^*_i, \sigma^*_{-i}) \geq u_i(s_i, \sigma^*_{-i})$ for all $s_i \in S_i$
- NE is such set of strategies, that no player is willing to deviate to any other pure strategy.
- In other words, the strategies are multilateral best responses

Nash Equilibrium -examples

- In the Ad game, NE is a pair of pure strategies (A, A)

		Player 2	
		A	N
Player 1	A	40, 40	60, 30
	N	30, 60	50, 50

Nash Equilibrium -examples

- In the game below, (M, R) is NE

		Player 2	
		L	*R*
	U	3, *1*	0, *2*
Player 1	M	0, *0*	3, *1*
	D	1, *2*	1, *1*

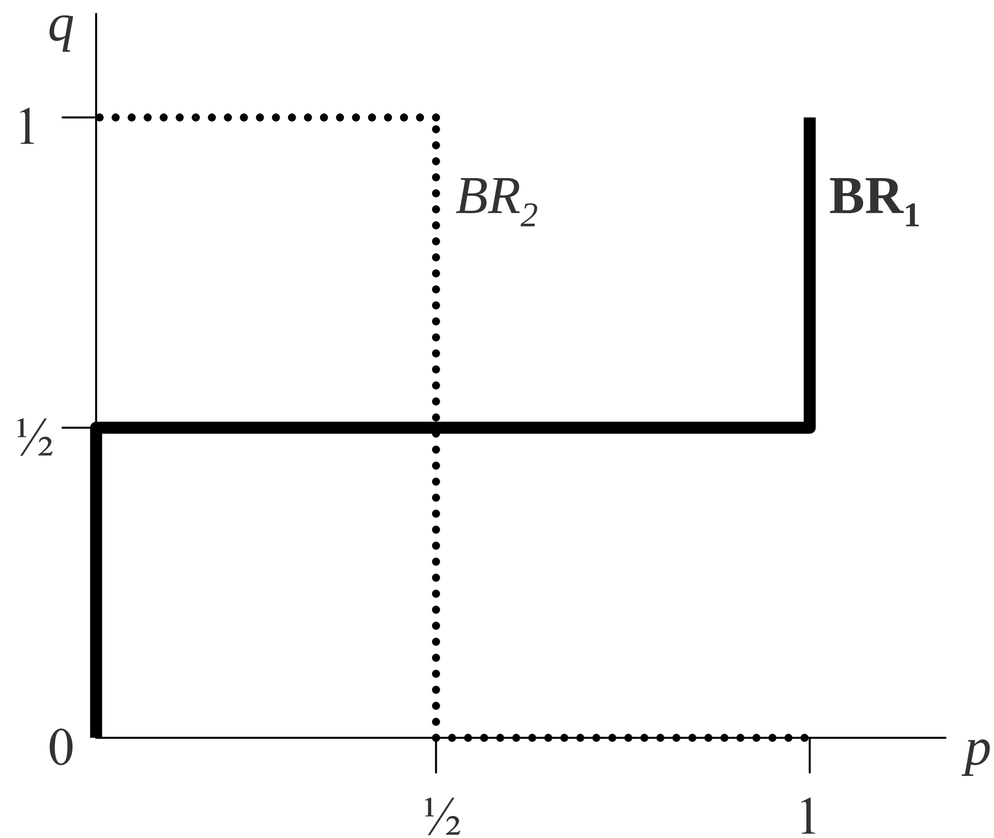
Nash Equilibrium -examples

- „Matching pennies” does not have a pure-strategy NE

		Pl. 2	
		Heads	Tails
Pl. 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

- Let *p* denote the prob. of Heads for player 1, *q* – the prob. of Heads for player 2

NE in mixed strategies



The BR_i are the best-response correspondences

An Easy Trick

- Drawing best response correspondences is fun, but time consuming.
- Instead, we can use the following fact:
  - If player *i* mixes between two or more pure strategies in NE, then in equilibrium she must be indifferent between all these pure strategies
- Hence, if we want to find a mixed-strategy NE, we must find the probability distribution, which will make player *i* indifferent between the pure strategies

Battle of Sexes

- Find all NE in the game below

		Wife	
		Football	Ballet
Husband	Football	<u>2</u>, <u>3</u>	0, 0
	Ballet	1, 1	<u>3</u>, <u>2</u>

Existence of Nash Equilibrium

- Nash Theorem:
  - Every finite normal form game has a mixed-strategy equilibrium
  - Proof (heuristic): Let $r_i(\sigma_{-i})$ denote the best-response correspondence for player i.
    Let $r(\sigma) : \Sigma \rightarrow \Sigma$ denote the Cartesian product of r_i's. Then notice that $r(\sigma)$ is nonempty and convex for all σ and has a closed graph. Then refer to Kakutani's fixed-point theorem and state that $r(\sigma)$ has a fixed point, where $\sigma^* \in r(\sigma^*)$, which is a NE.

Other Existence Theorems

- Debreu-Glicksberg-Fan Theorem:
  - Consider a normal-form game where all S_i are compact convex subsets of Euclidean space. If the payoff functions u_i are continuous in s and quasi-concave in s_i then there exists a pure-strategy NE.
- Glicksberg Theorem:
  - Consider a normal-form game where all S_i are compact subsets of metric space. If the payoff functions u_i are continuous then there exists a mixed-strategy NE.

Non-Uniqueness of NE

- The bigger problem is that there are often many NE. We can use „refinements” of the equilibrium to find a more concrete solution:
  - Focal Point
  - Pareto Perfection

Pareto Perfection

- (B,F) Pareto dominates (D,E)

	Player 2				
Player 1		E	F	G	H
	A	2, 2	2, 6	1, 4	0, 4
	B	0, 0	10, 10	2, 1	1, 1
	C	7, 0	2, 2	1, 5	5, 1
	D	9, 5	1, 3	0, 2	4, 4

Focal Point

- Which NE is the focal point?

		Player 2			
Player 1		E	F	G	H
	A	100, 100	2, 6	1, 4	0, 4
	B	0, 0	100, 100	2, 1	1, 1
	C	7, 0	2, 2	99, 99	5, 1
	D	9, 5	1, 3	0, 2	100, 100

Correlated Equilibria

- Look at the Battle of Sexes. Do you think that the mixed-strategy equilibrium would be played?
- Not if players can communicate before the play.Then they can avoid the bad outcomes. They can allow Nature (a commonly observed random event) to decide which equilibrium they will play. Their expected payoffs may be bigger than in the mixed-strategy NE.
- Interestingly, if the players can observe imperfectly correlated signals, they can sometimes achieve expected payoff **larger than** in any NE.